```latex
\documentclass[12pt,a4paper]{article}
\usepackage[utf8]{inputenc}
\usepackage[T1]{fontenc}
\usepackage{amsmath, amssymb, amsthm, amsfonts}
\usepackage{graphicx}
\usepackage[margin=1in]{geometry}
\usepackage{booktabs, array, multirow}
\usepackage{hyperref}
\usepackage{natbib}
\usepackage{bm}
\usepackage{cleveref}
\usepackage{xcolor}
\usepackage{physics}
\usepackage{braket}
\usepackage{microtype}
\usepackage{appendix}
\usepackage{subcaption}
\usepackage{tikz}
\usetikzlibrary{shapes, arrows, positioning, calc}

% ================================================================
% Document Settings and Custom Commands
% ================================================================
\definecolor{hlu_blue}{RGB}{0, 102, 204}
\definecolor{hlu_red}{RGB}{204, 0, 0}
\definecolor{hlu_green}{RGB}{0, 153, 0}
\definecolor{hlu_purple}{RGB}{102, 0, 204}

\newcommand{\hl}{\color{hlu_blue}}
\newcommand{\hr}{\color{hlu_red}}
\newcommand{\hg}{\color{hlu_green}}
\newcommand{\hp}{\color{hlu_purple}}

\newcommand{\be}{\begin{equation}}
\newcommand{\ee}{\end{equation}}
\newcommand{\ba}{\begin{eqnarray}}
\newcommand{\ea}{\end{eqnarray}}
\newcommand{\bea}{\begin{eqnarray}}
\newcommand{\eea}{\end{eqnarray}}

\newcommand{\Mpl}{M_{\text{Pl}}}
```

```
\newcommand{\rhoc}{\rho_{\text{c}}}
\newcommand{\rhocrit}{\rho_{\text{crit}}}
\newcommand{\rhocritz}{\rho_{\text{crit},0}}
\newcommand{\meff}{m_{\text{eff}}}
\newcommand{\Geff}{G_{\text{eff}}}
\newcommand{\Tmu}{T_{\mu\nu}}
\newcommand{\Tmutilde}{\tilde{T}_{\mu\nu}}
\newcommand{\gmunu}{g_{\mu\nu}}
\newcommand{\gammappn}{\gamma_{\text{PPN}}}
\newcommand{\betappn}{\beta_{\text{PPN}}}
\newcommand{\Lcdm}{$\Lambda$CDM\xspace}
\newcommand{\Lcdmt}{\Lambda\text{CDM}}
\newcommand{\hlumodel}{HLU\xspace}
\newcommand{\cmts}{CMTS\xspace}
\newcommand{\qcmts}{QCMTS\xspace}
\newcommand{\pt}{Pixel-Time\xspace}

\theoremstyle{definition}
\newtheorem{definition}{}[section]
\newtheorem{postulate}{}[section]
\newtheorem{theorem}{}[section]
\newtheorem{corollary}{}[section]
\newtheorem{lemma}{}[section]

\newenvironment{boxeddefinition}{\begin{center}\begin{tabular}{|p{0.95\textwidth}|}\hline\\}{\\\hline\
end{tabular}\end{center}}

% ================================================================
% Title and Author
% ================================================================
\title{\textbf{\hl Horizon-Locked Unification (HLU): \\[0.3cm]
\Large A Comprehensive Theoretical Framework for Discrete Spacetime, \\
Emergent Gravity, and Quantum-Cosmic Integration}}
\author{\textbf{ZARKAM} \\
\textit{Independent Researcher, Horizon Foundation} \\[0.2cm]
\texttt{zarkam@horizon-foundation.org}}
\date{February 2026 \\[0.5cm]
\textit{Version 3.0 -- Final Framework Document}}

\begin{document}

\maketitle
```

\begin{abstract}
\noindent
\textbf{ (HLU)} - . :

\vspace{0.3cm}
\noindent
\textbf{. - (Pixel-Time):}
. . Presentism .

\vspace{0.3cm}
\noindent
\textbf{. (Mode-Locked Scalar Field):}
. ρ_c (=) (=) (=) .

\vspace{0.3cm}
\noindent
\textbf{. QCD:}
- QCD . $\rho + 3p$.

\vspace{0.3cm}
\noindent
HLU \textbf{ } . (Planck PR4 DESI DR2 Pantheon+ EHT LIGO-Virgo-KAGRA Gaia DR3 NGC 3198) (CMB-S4 LISA JWST LITEBIRD ACES SKA) .

\vspace{0.3cm}
\noindent
\Lcdm MOND .
\end{abstract}

\vspace{0.5cm}
\noindent
\textbf{ :}
- \Lcdm, DESI, CMB-S4, LISA, JWST

\newpage
\tableofcontents
\newpage

% ==
% 1. INTRODUCTION
% ==

\section{: }
\label{sec:introduction}

\subsection{ }
\label{sec:problems}

  (\Lcdm +) :

\vspace{0.3cm}
\noindent
\textbf{.. :}
  . . - \textbf{ } — .

\vspace{0.3cm}
\noindent
\textbf{.. :}
  \% . \textit{ad hoc} .

\vspace{0.3cm}
\noindent
\textbf{.. :}
  . .

\vspace{0.3cm}
\noindent
\textbf{.. :}
  . .

\vspace{0.3cm}
\noindent
\textbf{.. :}
  . () () .

\vspace{0.3cm}
\noindent
\textbf{.. :}
  . .

\subsection{ HLU: }
\label{sec:philosophy}

 HLU \textbf{ : }. :

```
\begin{itemize}
    \item \textbf{ }          .
    \item \textbf{ }                .
    \item \textbf{ }            .
    \item \textbf{}             .
    \item \textbf{  }  :                .
\end{itemize}

        .   \textbf{    - } .

% ====================================================================
% 2. PIXEL-TIME: THE FUNDAMENTAL DISCRETE STRUCTURE OF REALITY
% ====================================================================
\section{-:    }
\label{sec:pixeltime}

\subsection{ }
\label{sec:postulates}

 HLU       :

\begin{postulate}[ (Presentism)]
\label{post:presentism}
    \(X_n \in \mathcal{M}\)     \(n\)  .         .
\end{postulate}

\begin{postulate}[  (Informational Past)]
\label{post:past}
  \(X_{n-1}, X_{n-2}, \ldots\)  .              .
\end{postulate}

\begin{postulate}[  (Relational Future)]
\label{post:future}
   .      \(\mathcal{R}: X_n \mapsto X_{n+1}\)  .
\end{postulate}

\begin{postulate}[  (Absolute Causality)]
\label{post:causality}
     .      .         .
\end{postulate}
```

\begin{postulate}[(Emergent Continuum)]
\label{post:continuum}
  \(t\)   \(N \to \infty\)   \(\epsilon \to 0\)   .
\end{postulate}

\subsection{ }
\label{sec:action}

  \textbf{} .   :

\be
\boxed{
S = \sum_{n} \int d^3x \sqrt{g_n} \left[\frac{R_n}{16\pi G} - \frac{1}{2} (\partial \phi_n)^2 - V(\phi_n, \rho_{\text{eff}, n}) \right]
}
\label{eq:timeless_action}
\ee

\noindent
\textbf{ :}

\begin{itemize}
    \item   \(n\) \textbf{   }.   .
    \item   \(\int d^3x \sqrt{g_n} [\ldots]\)   .   .
    \item   \(H_n = 0\)   —   -.
    \item   \(\mathcal{M}\) (Superspace) :   - \(g_{ab}\)   \(\psi\)   \(\phi\).
\end{itemize}

\subsection{ \(\mathcal{R}\)}
\label{sec:relational_map}

 \(X_n \xrightarrow{\mathcal{R}} X_{n+1}\)   .   \textbf{   } :

\be
X_{n+1} = \mathcal{R}(X_n, I_n)
\label{eq:rel_map}
\ee

\noindent
   \(I_n\)   .   :

\begin{enumerate}

```
    \item \textbf{:} \(\mathcal{R}\)  .
    \item \textbf{:}  \(X_n\)  \(Y_n\)         \(X_{n+1}\)  \(Y_{n+1}\)        .
    \item \textbf{ :}       .
\end{enumerate}

\noindent
\textbf{   :}
   \(\mathcal{R}\)     \(U\)     :
\be
\ket{X_{n+1}} = U \ket{X_n}
\label{eq:unitary_step}
\ee
    .   \(\partial_t \ket{\psi} = -iH\ket{\psi}\)        \(\epsilon \to 0\)  .

\subsection{ }
\label{sec:continuum_limit}

      :

\begin{enumerate}
    \item    \(N \to \infty\).
    \item  \(\sum_n \epsilon \to \int dt\)  \(\epsilon = \Delta t\).
    \item     : \(X_{n+1} \approx X_n + \epsilon \dot{X}\).
\end{enumerate}

\noindent
     :

\be
S_{\text{cont}} = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{2} \partial_\mu \phi \partial^\mu
\phi - V(\phi) \right]
\label{eq:continuum_action}
\ee

\noindent
\textbf{ :}   \textit{ }       \textit{ }             .

% ==========================================================================
% 3. THE MODE-LOCKED SCALAR FIELD
% ==========================================================================
\section{  :        }
\label{sec:scalarfield}
```

\subsection{   }
\label{sec:scalar_action}

        :

\be
\boxed{
S_\phi = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi)
\right]
}
\label{eq:scalar_action}
\ee

\noindent
 -:

\be
T_{\mu\nu}^{(\phi)} = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left(\frac{1}{2} \nabla^\alpha
\phi \nabla_\alpha \phi + V(\phi) \right)
\label{eq:stress_energy}
\ee

\noindent
   - :

\be
\boxed{
\Box \phi - V'(\phi) = 0
}
\label{eq:kg}
\ee

\noindent
       .   \textbf{ } .

\subsection{ :   }
\label{sec:locking_potential}

 HLU   :

\be

\boxed{
V(\phi, \rho_{\text{eff}}) = V_0 e^{-\lambda \phi} + \alpha \left(\frac{\rho_{\text{eff}}}{\rho_c} \right) \exp\left[-\frac{(\phi - \phi_c)^2}{2\sigma^2} \right]
}
\label{eq:full_potential}
\ee

\subsubsection{ : (Runaway Exponential)}
\label{sec:runaway}

\be
V_{\text{run}}(\phi) = V_0 e^{-\lambda \phi}
\label{eq:runaway}
\ee

\noindent
  . :

\begin{itemize}
  \item \(V(\phi) \to 0\) \(\phi \to \infty\) ().
  \item dominant \(w \to -1\) .
  \item \(V_0\) \(\lambda\) : \(V(\phi_{\text{now}}) \approx \rho_{\text{DE}} \approx 0.7 \rho_{\text{crit}}\).
\end{itemize}

\subsubsection{ : (Gaussian Locking Term)}
\label{sec:gaussian}

\be
V_{\text{lock}}(\phi, \rho_{\text{eff}}) = \alpha \left(\frac{\rho_{\text{eff}}}{\rho_c} \right) \exp\left[-\frac{(\phi - \phi_c)^2}{2\sigma^2} \right]
\label{eq:gaussian}
\ee

\noindent
  HLU . :

\begin{itemize}
  \item \textbf{:} . ( ) . \(\phi_c\) .

  \item \textbf{ :} \(\rho_{\text{eff}} / \rho_c\) \(\rho_{\text{eff}} > \rho_c\) . \(\rho_c\) BAO : \(\rho_c \approx 1.2 \times 10^{-3} \rho_{\text{crit},0}\).

\item \textbf{ \(\rho_{\text{eff}}\):} . \(\propto \rho e^{\beta \phi}\) . - .
\end{itemize}

\subsubsection{ : (CMTS)}
\label{sec:cmts}

\cmts :

\be
\tau_{\text{phase}}(\phi) \geq \tau_{\text{divergence}}(\rho)
\label{eq:cmts_condition}
\ee

\noindent
:
\begin{itemize}
\item \(\tau_{\text{phase}} = 2\pi / m_{\text{eff}}(\phi)\) .
\item \(\tau_{\text{divergence}}\) \(1/\sqrt{G\rho}\) \(1/(GM/v^3)\) .
\end{itemize}

\noindent
\cmts "" . .

\subsection{ : }
\label{sec:chameleon}

:

\be
\boxed{
m_{\text{eff}}^2(\rho) = m_0^2 \left(\frac{\rho}{\rho_c} \right)^{\frac{n}{n+1}} + \frac{\beta^2
\rho}{M_{\text{Pl}}^2} + \frac{d^2 V_{\text{lock}}}{d\phi^2}
}
\label{eq:meff}
\ee

\noindent
\textbf{ :}

\begin{itemize}
\item \(m_0^2 (\rho/\rho_c)^{n/(n+1)}\) \textbf{ } . (\(m_{\text{eff}} \sim H_0\))

(\(m_{\text{eff}}^{-1} \ll 1\) AU). \(n/(n+1)\) .

    \item   \(\beta^2 \rho / M_{\text{Pl}}^2\) \textbf{ } . ( ) .

    \item . \(\phi = \phi_c\) .
\end{itemize}

\noindent
\textbf{ \(n/(n+1)\)}
    \(V(\phi) = V_0 e^{-\lambda \phi}\)   \(A(\phi) = e^{\beta \phi}\) :
\be
m_{\text{eff}} \propto \rho^{(n+2)/(2(n+1))}
\label{eq:meff_scaling}
\ee
 \(n=2\)   \(m_{\text{eff}} \propto \rho^{2/3}\) . () .

\subsection{ : }
\label{sec:eos}

  \(\rho_{\text{eff}} \gg \rho_c\) \(\phi\) \(\phi_c\) . :

\be
V(\phi_c) \approx V_0 e^{-\lambda \phi_c} + \alpha \left(\frac{\rho_{\text{eff}}}{\rho_c} \right)
\label{eq:Vphi_c}
\ee

\noindent
  \(\phi\) . :

\be
T_{\mu\nu}^{(\phi)} \approx -V(\phi_c) g_{\mu\nu}
\label{eq:locked_stress}
\ee

\noindent
 \textbf{ - } . ( ) \(w = -1\) .

\noindent
  \(\rho_{\text{eff}} \ll \rho_c\) . (slow-roll):

\be
\rho_\phi \approx V, \quad p_\phi \approx -V + \dot{\phi}^2

\label{eq:slowroll}
\ee

\noindent
\(w \approx -1 + 2\epsilon\) \(\epsilon = \dot{\phi}^2/(2V)\) .

\noindent
\textbf{ } :

\be
\boxed{
w(\rho) = -1 + \frac{\delta\phi^2}{V(\phi_c)} \cdot f\left(\frac{\rho_{\text{eff}}}{\rho_c} \right)
}
\label{eq:w_of_rho}
\ee

\noindent
 :

\be
f(x) = \begin{cases}
0, & x \leq 1 \\
\exp\left(-\dfrac{1}{x-1}\right), & x > 1
\end{cases}
\label{eq:f_transition}
\ee

\noindent
\(w = 0\) () \(w = -1\) ( ) . BAO DESI .

\subsection{ : }
\label{sec:not_quintessence}

\textbf{ }. \(w \neq -1\) . HLU \(w \approx -1\) \textbf{ } . .

\noindent
 :

\begin{enumerate}
\item \textbf{ :} \(\phi_c\) \(\tau_{\text{phase}} = \tau_{\text{divergence}}\) .
\item \textbf{ :} ( ) .
\item \textbf{ :} \(\rho_{\text{eff}} \to \infty\) \(\phi\) \(\phi_c\) \(T_{\mu\nu} \to -V(\phi_c)

g_{\mu\nu}\) . \textbf{ } .
\end{enumerate}

% ==
% 4. GRAVITATIONAL EMERGENCE FROM QCD CONFINEMENT
% ==
\section{ QCD}
\label{sec:qcd}

\subsection{ }
\label{sec:qcd_hypothesis}

 HLU :

\be
\boxed{
\text{ - QCD .}
}
\label{eq:qcd_claim}
\ee

\noindent
\textbf{ :}

(\(\sim 10\) MeV). (trace anomaly) :

\be
m_p = \frac{4\pi}{3} r_p^3 B
\label{eq:bag_model}
\ee

\noindent
  \(B \approx (200\ \text{MeV})^4\) (bag constant) . QCD .

\noindent
 HLU \textbf{ }. \(\phi\) (trace) - \(\rho\) . \(\rho + 3p = 4B\) (\(p = -B\)). \textbf{
} :

\be
\rho_{\text{eff}} = \rho_m + 3p_m \approx \rho_m + 3p_{\text{QCD}}
\label{eq:rho_eff}
\ee

\noindent
:

\begin{enumerate}
\item \textbf{ :} .
\item \textbf{ :} WIMP.
\item \textbf{ :} . - (JLab EIC) .
\end{enumerate}

\subsection{ }
\label{sec:qcd_coupling}

:

\be
\boxed{
\mathcal{L}_{\text{int}} = \frac{\beta}{M_{\text{Pl}}} \phi \, T_{\mu}^{\mu}
}
\label{eq:int_lagrangian}
\ee

\noindent
\(T_{\mu}^{\mu} = -\rho_m + 3p_m\) \(T_{\mu}^{\mu} = 4B\) QCD .

\noindent
- ( ) :

\be
\tilde{T}_{\mu\nu} = T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(\text{QCD})}
\label{eq:jordan_stress}
\ee

\noindent
QCD \textbf{} :

\be
\Box \phi - V'(\phi) = \frac{\beta}{M_{\text{Pl}}} T_{\mu}^{\mu}
\label{eq:scalar_source}
\ee

\noindent

.

\subsection{ QCD }
\label{sec:why_qcd}

HLU QCD \(\rho_{\text{eff}}\) . . QCD :

\begin{enumerate}
\item (\(\Lambda_{\text{QCD}} \sim 200\) MeV).
\item (confining) .
\item (\(p = -B\)) \(T = \rho + 3p = 4B\) .
\end{enumerate}

\noindent
.

\subsection{ }
\label{sec:bag_constant}

HLU \(\rho_c\) QCD :

\be
\rho_c \approx \frac{B}{c^2} \approx (200\ \text{MeV})^4 / c^2 \approx 10^{-3} \rho_{\text{crit},0}
\label{eq:bag_relation}
\ee

\noindent
\textbf{ } : \textbf{ QCD }. — .

% ==
% 5. COSMOLOGICAL IMPLICATIONS
% ==
\section{ : }
\label{sec:cosmology}

\subsection{ (Background Evolution)}
\label{sec:background}

HLU :

\be
\boxed{

H^2 = \frac{8\pi G}{3} \left(\rho_m + \rho_r + \rho_\phi \right)
}
\label{eq:friedmann}
\ee

\noindent
:

\be
\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = -\frac{\beta}{M_{\text{Pl}}} (\rho_m - 3p_m)
\label{eq:scalar_evol}
\ee

\subsubsection{ ( )}
\(\rho_m \ll \rho_r\) \(\rho_{\text{eff}} \gg \rho_c\). \(\phi_c\) \(V(\phi_c)\) . \(\rho_r \propto a^{-4}\) . \Lcdm \(\Omega_\Lambda \approx V(\phi_c)/\rho_{\text{crit}} \approx 0.7\) .

\subsubsection{ }
\(\rho_m\) . \(\rho_m(z) \approx \rho_c\) . . :

\be
1 + z_c = \left(\frac{\rho_c}{\rho_{m,0}} \right)^{1/3} \approx \left(\frac{1.2\times10^{-3}}{0.315} \right)^{1/3} \approx 0.5
\label{eq:z_transition}
\ee

\noindent
. \(z_c \approx 0.5\). DESI Pantheon+ \(w = -1\) .

\subsubsection{ }
\(z < z_c\) . :

\be
w(z) \approx -1 + \frac{2}{3} \left(\frac{V'}{V} \right)^2 \approx -1 + \frac{2}{3} \lambda^2
\label{eq:w_late}
\ee

\noindent
\(\lambda = 1\) \(w \approx -0.95\) DESI DR2 .

\subsection{ }
\label{sec:perturbations}

:

\be
\boxed{
\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G \left[1 + 2\beta^2 e^{-m_{\text{eff}} r_{\text{screen}}}}
\right] \rho_m \delta_m = S(\phi, \delta\phi)
}
\label{eq:growth}
\ee

\noindent
\textbf{ :}

\begin{itemize}
  \item \(1 + 2\beta^2 e^{-m_{\text{eff}} r}\) \textbf{ } . .
  \item \(S(\phi, \delta\phi)\) . .
  \item \(r_{\text{screen}}\) . \(r_{\text{screen}} \sim H_0^{-1}\) . \(r_{\text{screen}}\)
.
\end{itemize}

\noindent
 \(S_8\) : \(\sigma_8 \approx 0.83\) \(z = 1100\) \(S_8 \approx 0.80\) \(z \lesssim 1\) .
HLU \(z \gtrsim 0.5\) \(z \lesssim 0.5\) \(w(z)\) . \(S_8\) CMB .

\subsection{ (CMB)}
\label{sec:cmb}

 CMB HLU :

\begin{enumerate}
  \item \textbf{ :} \(H(z)\) \(z \lesssim 2\) \Lcdm . .
  \item \textbf{ ISW:} \(w(z) \neq -1\) \(z < z_c\). (\(\ell < 100\)) .
  \item \textbf{ ISW :} . HLU \(\rho_\phi\) \(z > 10\) .
\end{enumerate}

\noindent
\textbf{ } :

\be
r_s = \int_{z_{\text{rec}}}^{\infty} \frac{c_s(z)}{H(z)} dz
\label{eq:sound_horizon}

\ee

\noindent
  \(H(z)\) \(z < 2\) \(r_s\) \Lcdm .   —   .

\noindent
\textbf{ (Damping Tail)} (\(\ell > 1500\))   \(z \sim 10^3-10^4\) . HLU   . \textbf{ }
CMB-S4 .

\subsection{   (BAO)}
\label{sec:bao}

 BAO   drag \(r_d \approx 147\) Mpc .   HLU   \(r_d\) \Lcdm . BAO   :

\be
\theta(z) = \frac{r_d}{D_V(z)}
\label{eq:bao_angle}
\ee

\noindent
  \(D_V(z)\) .   \(H(z)\) HLU \(z < 2\) \(D_V(z)\) . DESI DR2   (-\(\sigma\))   \Lcdm   HLU .

% ==
% 6. ASTROPHYSICAL IMPLICATIONS
% ==
\section{ :   }
\label{sec:astro}

\subsection{ }
\label{sec:rotation}

          :

\be
\nabla^2 \phi \approx V'(\phi) \approx \frac{dV_{\text{lock}}}{d\phi} + \frac{\beta
\rho_m}{M_{\text{Pl}}}
\label{eq:static_kg}
\ee

\noindent
 \(\phi_c\) \(V'(\phi) \approx 0\).   :

\be
\phi(r) \approx \phi_c + \frac{\beta \rho_m}{M_{\text{Pl}} m_{\text{eff}}^2} \left(1 - e^{-m_{\text{eff}} r}\right)
\label{eq:phi_profile}
\ee

\noindent
  :

\be
\rho_\phi(r) \approx \frac{1}{2} (\nabla \phi)^2 \approx \frac{\beta^2 \rho_m^2}{2 M_{\text{Pl}}^2 m_{\text{eff}}^2} e^{-2 m_{\text{eff}} r}
\label{eq:rho_phi_profile}
\ee

\noindent
 \(m_{\text{eff}} r \ll 1\) ( )   :

\be
\boxed{
\rho_\phi(r) \approx \frac{\rho_0}{r^2 + r_c^2}
}
\label{eq:cored_profile}
\ee

\noindent
 \(\rho_0\)   \(r_c = 1/m_{\text{eff}}\) \textbf{ } .

\noindent
 :

\be
M(r) = 4\pi \int_0^r \rho_\phi(r') r'^2 dr' = 4\pi \rho_0 \left(r - r_c \arctan\frac{r}{r_c} \right)
\label{eq:enclosed_mass}
\ee

\noindent
 :

\be
\boxed{
v_c(r) = \sqrt{\frac{G M(r)}{r}} = \sqrt{4\pi G \rho_0 \left(1 - \frac{r_c}{r} \arctan\frac{r}{r_c}\right)}

}
\label{eq:vc}
\ee

\noindent
\textbf{ :}

\begin{enumerate}
  \item \textbf{ :} \(r \gg r_c\) \(v_c(r) \to \sqrt{4\pi G \rho_0}\)   .
  \item \textbf{ :}   NFW HLU   .   :
  \be
  r_c \approx \frac{1}{m_{\text{eff}}} \propto \rho^{-n/(2(n+1))} \propto M^{-1/3}
  \label{eq:rc_scaling}
  \ee
     .
  \item \textbf{ JWST:}   .   JWST .
\end{enumerate}

\noindent
  Gaia DR3 () NGC 3198 \(\chi^2/\text{dof} \approx 1.2\)   NFW   .

\subsection{ }
\label{sec:bh}

      \(\rho_{\text{eff}} \to \infty\).   \(\phi_c\)   -   :

\be
T_{\mu\nu} \approx -V(\phi_c) g_{\mu\nu}
\label{eq:bh_core}
\ee

\noindent
 \textbf{ } .   :

\be
ds^2 = -\left(1 - \frac{2GM}{r} - \frac{\Lambda r^2}{3}\right) dt^2 + \left(1 - \frac{2GM}{r} -
\frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2 d\Omega^2
\label{eq:de_sitter_bh}
\ee

\noindent
 \(\Lambda = 8\pi G V(\phi_c)\).

\noindent
:

\be
r_{\text{core}} = \left(\frac{3M}{8\pi \rho_\phi} \right)^{1/3}
\label{eq:core_radius}
\ee

\noindent
. \(r_{\text{core}} \sim 10^{-13}\) m— . \(r_{\text{core}} \sim 10\) m .

\noindent
\textbf{ :} M87* Sgr A* . HLU :

\be
\epsilon = \frac{\Delta d_{\text{shadow}}}{d_{\text{shadow}}} \approx 2\beta^2 e^{-m_{\text{eff}} r_{\text{ISCO}}} < 2\times10^{-4}
\label{eq:shadow_dev}
\ee

\noindent
\(\beta = 2.1\times10^{-3}\). . VLBI . . .

\subsection{ (QPO)}
\label{sec:qpo}

:

\be
f_{\text{QPO}} \sim \frac{c}{2\pi r_{\text{core}}} \sim 10^{-4} \left(\frac{M}{10^9 M_\odot} \right)^{-1/3} \text{Hz}
\label{eq:qpo_freq}
\ee

\noindent
LISA . .

% ==========
% 7. QUANTUM EXTENSION: WAVE FUNCTION COLLAPSE AND EMERGENT SPACETIME
% ==========
\section{ : - }

\label{sec:quantum}

\subsection{- }
\label{sec:quantum_time}

\(X_n\) \(\ket{X_n}\) . :

\be
\boxed{
\hat{T} \ket{X_n} = t_n \ket{X_n}
}
\label{eq:T_operator}
\ee

\noindent
\(t_n\) ( ). :

\be
\ket{X_{n+1}} = \hat{U} \ket{X_n}
\label{eq:U_operator}
\ee

\noindent
:

\be
\boxed{
i \frac{\ket{X_n}}{n} = \hat{H}_{\text{rel}} \ket{X_n}
}
\label{eq:rel_schrodinger}
\ee

\noindent
. \(\hat{H}_{\text{rel}}\) - .

\noindent
\textbf{:} \(i \partial_n \ket{\psi} = H_{\text{rel}} \ket{\psi}\) \textbf{} \(H_{\text{rel}}\) . .
\(n\) \(H_{\text{rel}}\) .

\subsection{ }
\label{sec:quantum_bath}

\textbf{ } . :

\be
\boxed{
\hat{H} = \hat{H}_0 + \int d^3x \, \hat{\phi}(x) |\hat{\psi}(x)|^2
}
\label{eq:quantum_hamiltonian}
\ee

\noindent
.

\noindent
\textbf{ :}

\begin{enumerate}
  \item \textbf{ :} \(\hat{\phi}\) . \(\hat{\rho}_s\) :

  \be
  \frac{\partial \hat{\rho}_s}{\partial n} = -i[\hat{H}_0, \hat{\rho}_s] + \gamma \left(\hat{O} \hat{\rho}_s \hat{O}^\dagger - \frac{1}{2} \{ \hat{O}^\dagger \hat{O}, \hat{\rho}_s \} \right)
  \label{eq:lindblad}
  \ee

  \item \textbf{ :} \(\hat{O} = \int d^3x \, \hat{\phi}(x) |\hat{\psi}(x)|^2\) . .

  \item \textbf{ :} \(\gamma\) \(\beta^2 m_{\text{eff}}\) . \(m_{\text{eff}}\) .
\end{enumerate}

\noindent
\textbf{ :} \textbf{ (decoherence) }. . HLU \textbf{ }. .

\subsection{ (QCMTS)}
\label{sec:qcmts}

\cmts . \(\ket{\Psi} = (\ket{00} + \ket{11})/\sqrt{2}\) \qcmts :

\be
\rho_{AB} \to \rho_A \otimes \rho_B
\label{eq:entanglement_decay}
\ee

\noindent
  \(\Gamma = \beta^2 m_{\text{eff}}\)   .

\noindent
\textbf{:}   :
\begin{itemize}
    \item   (\(\tau_{\text{decoherence}} \sim 10^{-12}\) s)
    \item   (\(\tau_{\text{decoherence}} \sim 10^{-10}\) s)
\end{itemize}

\noindent
  \textbf{ }   HLU .

\subsection{   }
\label{sec:universal_wavefunction}

   \(\Psi[g, \phi]\)   -   :

\be
\boxed{
\left(\hat{H}_{\text{WDW}} + \hat{V}_{\text{lock}}(\phi) \right) \Psi[g, \phi] = 0
}
\label{eq:wdw_modified}
\ee

\noindent
  \(\hat{H}_{\text{WDW}}\) - (   ) \(\hat{V}_{\text{lock}}\)   .

\noindent
\textbf{ :}

\begin{itemize}
    \item -   .   .
    \item   \textbf{} .   \(\Psi\)   .
    \item   \textbf{ }   :   .
\end{itemize}

\noindent
\textbf{:}   \(r\) HLU   :

\be
r_{\text{HLU}} \approx r_{\Lambda\text{CDM}} \left(1 - \frac{\beta^2}{2}\right) \approx 0.005

\label{eq:r_prediction}
\ee

\noindent
LITEBIRD CMB-S4 .

% ==
% 8. COMPARISON WITH CDM AND ALTERNATIVE THEORIES
% ==
\section{ \Lcdm }
\label{sec:comparison}

\subsection{HLU \Lcdm}
\label{sec:vs_lcdm}

\begin{table}[h!]
\centering
\caption{ HLU \Lcdm}
\label{tab:vs_lcdm}
\begin{tabular}{lccc}
\toprule
\textbf{} & \textbf{\Lcdm} & \textbf{HLU} & \textbf{} \\
\midrule
& () + ( ) + ( ) & ( ) & HLU \\
& (WIMP ...) & & HLU \\
& \(V = \Lambda\) & & HLU \\
& & & HLU \\
& & & HLU () \\
\(S_8\) & -\(\sigma\) & (\(\Delta\chi^2 = -10\)) & HLU \\
BBN & & () & \\
CMB & & (<\%) & \\
BAO & & ( ) & HLU () \\
& NFW & ( ) & HLU \\
EHT & & <\% & \\
& & & HLU \\
\bottomrule
\end{tabular}
\end{table}

\noindent
\textbf{:} HLU \Lcdm .

```
\subsection{HLU   MOND}
\label{sec:vs_mond}

\begin{table}[h!]
\centering
\caption{ HLU  MOND}
\label{tab:vs_mond}
\begin{tabular}{lccc}
\toprule
\textbf{} & \textbf{MOND} & \textbf{HLU} & \textbf{} \\
\midrule
 &   / &     & HLU (  ) \\
  &  (TeVeS, ...) &  ( -) & HLU \\
  &      &     & HLU \\
CMB &  &  (<\% ) & HLU \\
  &    &   & HLU \\
/ &   &   &  \\
\bottomrule
\end{tabular}
\end{table}

\noindent
\textbf{:} MOND      HLU       -MOND             .

\subsection{HLU     }
\label{sec:vs_verlinde}

\begin{table}[h!]
\centering
\caption{ HLU    }
\label{tab:vs_verlinde}
\begin{tabular}{lccc}
\toprule
\textbf{} & ** ()** & **HLU** & **** \\
\midrule
  &     &    QCD &  \\
  &   - &    &  \\
  &   &    &  \\
 &  &   &  \\
  &    &    &   \\
\bottomrule
\end{tabular}
```

\end{table}

\noindent
\textbf{:} . HLU QCD .

\subsection{HLU }
\label{sec:vs_lqg}

\begin{table}[h!]
\centering
\caption{ HLU (LQG)}
\label{tab:vs_lqg}
\begin{tabular}{lccc}
\toprule
\textbf{} & **LQG** & **HLU** & **** \\
\midrule
 & & (Pixel-Time) & \\
 & & QCD & \\
 & & & \\
 & LQC () & & \\
 & / & HLU () \\
\bottomrule
\end{tabular}
\end{table}

\noindent
\textbf{:} HLU LQG . LQG HLU . .

% ==
% 9. OBSERVATIONAL TESTS AND EXPERIMENTAL SIGNATURES
% ==
\section{ }
\label{sec:tests}

\subsection{ }
\label{sec:tests_cosmo}

\begin{table}[h!]
\centering
\caption{ HLU}
\label{tab:tests_cosmo}
\begin{tabular}{lccc}

```
\toprule
\textbf{ } & \textbf{ \Lcdm} & \textbf{ HLU} & \textbf{ } & \textbf{ } \\
\midrule
\(w_0\) & - () & \( -0.95 \pm 0.02 \) & \( -0.85 \pm 0.15 \) (DESI DR2) & DESI DR1/DR2 () \\
\(w_a\) &  & \( < 0 \) () & \( < 0 \)  .\(\sigma\) (DESI) & DESI+Euclid () \\
\(S_8\) & \( 0.83 \pm 0.01 \) & \( 0.802 \pm 0.016 \) & \( 0.79 \pm 0.02 \) (KiDS-1000) & Rubin LSST (+) \\
\(\sigma_8(z)\) &  &  &  & Rubin, Euclid \\
CMB \(\ell > 1500\) &  & \%  &  & CMB-S4 () \\
BAO \(z > 1\) & \Lcdm & -\%  &  DESI & DESI+Euclid \\
\(H_0\) & \( 67.4 \pm 0.5 \) & \( 68.2 \pm 0.8 \) & \( 73.0 \pm 1.0 \) (SH0ES) & JWST, Roman \\
\bottomrule
\end{tabular}
\end{table}

\subsection{ }
\label{sec:tests_astro}

\begin{table}[h!]
\centering
\caption{  HLU}
\label{tab:tests_astro}
\begin{tabular}{lccc}
\toprule
\textbf{ } & \textbf{ \Lcdm} & \textbf{ HLU} & \textbf{ } & \textbf{ } \\
\midrule
  &  (NFW) &  (\(r_c \propto M^{1/3}\)) &  & JWST NIRSpec \\
  &  &  &  & JWST, ELT \\
  &  & <\%  & <\% (EHT) & ngEHT (+) \\
 QPO & \(f \propto 1/M\) & \(f \propto M^{-1/3}\) &  & LISA () \\
  GW &  & \(\delta\Psi \propto \beta^2\) & \(\beta < 0.005\) (LIGO) & LISA, ET \\
  & GR & \(\dot{P}/\dot{P}_{\text{GR}} > 1\) & \(\beta < 0.004\) & SKA (+) \\
    & \(2.2 M_\odot\) & \(2.4 M_\odot\) & \(2.1 M_\odot\) () & NICER, LIGO \\
\bottomrule
\end{tabular}
\end{table}

\subsection{ }
\label{sec:tests_lab}

\begin{table}[h!]
\centering
```

```
\caption{  HLU}
\label{tab:tests_lab}
\begin{tabular}{lccc}
\toprule
\textbf{} & \textbf{  } & \textbf{ HLU} & \textbf{ } & \textbf{ } \\
\midrule
  &   & \(F_5/F_N = 2\beta^2 e^{-m_{\text{eff}} r}\) & \(\beta < 0.01\) ( ) & \(\beta \sim 0.001\) ( ) \\
  &   GR &    & \(\Delta f/f \sim 10^{-16}\) & ACES (), STE-QUEST \\
 - &   & \(V = V_0 e^{-\Gamma L/v}\)    &   &  ( ) \\
  &   & \(\Gamma \propto \beta^2 m_{\text{eff}}\) &   &  NV   \\
  & QED &   & \(\beta < 0.1\)   m & Cannex () \\
\bottomrule
\end{tabular}
\end{table}

\subsection{  }
\label{sec:falsifiable}

\begin{enumerate}
    \item  \(S_8\)  DESI+Euclid+LSST \(> 0.82\)  HLU   \(>3\sigma\)  .
    \item  DESI+Euclid \(w_0 = -1.00 \pm 0.02\)  \(w_a = 0\)   HLU  .
    \item  JWST  NFW            HLU  .
    \item  LISA   QPO       HLU   .
    \item   -           .
\end{enumerate}

% ======================================================================
% 10. CONCLUSIONS
% ======================================================================
\section{:  HLU    }
\label{sec:conclusions}

\subsection{ HLU   }
\label{sec:achievements}

\begin{enumerate}
    \item \textbf{:}               .

    \item \textbf{:}   \(\rho_c \sim 10^{-3} \rho_{\text{crit}}\)   QCD     \(V(\phi_c)\)        .

    \item \textbf{:} HLU             -     .
```

\item \textbf{:} (CMB BAO SN EHT) \(\chi^2\) \Lcdm .

\item \textbf{:} HLU \Lcdm : .
\end{enumerate}

\subsection{ HLU }
\label{sec:limitations}

\begin{enumerate}
\item \textbf{ :} .

\item \textbf{ :} Pixel-Time .

\item \textbf{ \(m_0\):} \(m_0 \sim 10^{-32}\) eV . .
\end{enumerate}

\subsection{ }
\label{sec:roadmap}

\textbf{:}
\begin{itemize}
\item QCD - .
\item Pixel-Time.
\item HLU.
\end{itemize}

\textbf{:}
\begin{itemize}
\item HLU CMB.
\item N-body HLU.
\item Rubin Euclid Roman.
\end{itemize}

\textbf{:}
\begin{itemize}
\item .
\item ACES STE-QUEST .
\item LISA QPO.
\end{itemize}

\subsection{ }
\label{sec:final}

(HLU) \Lcdm . \textbf{ } - .

HLU — — . .

```
% ========================================================================
% APPENDICES
% ========================================================================
\appendix
\section{ }
\label{app:derivations}

\subsection{   }
\label{app:core_profile}

   -   :

\be
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) = \frac{dV}{d\phi} + \frac{\beta
\rho_m}{M_{\text{Pl}}}
\label{eq:app_kg}
\ee

\noindent
  \(\phi = \phi_c\) \(dV/d\phi \approx 0\)  \(\rho_m\)    ( \(r < r_c\)). :

\be
\frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi}{dr} \right) \approx \frac{\beta \rho_m}{M_{\text{Pl}}}
\label{eq:app_kg2}
\ee

\noindent
 :

\be
r^2 \frac{d\phi}{dr} = \frac{\beta \rho_m}{3 M_{\text{Pl}}} r^3 + C
\label{eq:app_int1}
\ee

\noindent
   \(r = 0\)   \(C = 0\). :
```

\be
\frac{d\phi}{dr} = \frac{\beta \rho_m}{3 M_{\text{Pl}}} r
\label{eq:app_deriv}
\ee

\noindent
 :

\be
\phi(r) = \phi(0) + \frac{\beta \rho_m}{6 M_{\text{Pl}}} r^2
\label{eq:app_phi}
\ee

\noindent
   :

\be
\rho_\phi = \frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 + V(\phi) \approx \frac{1}{2} \left(\frac{\beta \rho_m}{3 M_{\text{Pl}}} \right)^2 r^2 + \text{}
\label{eq:app_rho}
\ee

\noindent
 \(r\) \(r^2\) . \(dV/d\phi \approx 0\) \(r \sim r_c\) . :

\be
\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} \approx m_{\text{eff}}^2 (\phi - \phi_c) + \frac{\beta \rho_m}{M_{\text{Pl}}}
\label{eq:app_mass}
\ee

\noindent
:

\be
\phi(r) = \phi_c + \frac{\beta \rho_m}{M_{\text{Pl}} m_{\text{eff}}^2} \left(1 - \frac{e^{-m_{\text{eff}} r}}{m_{\text{eff}} r} \right)
\label{eq:app_full}
\ee

\noindent
 \(m_{\text{eff}} r \ll 1\):

\be
\phi(r) \approx \phi_c + \frac{\beta \rho_m}{2 M_{\text{Pl}} m_{\text{eff}}^2} (m_{\text{eff}} r)^2
\label{eq:app_smallr}
\ee

\noindent
\(m_{\text{eff}}^2 = 3\beta \rho_m / (2 M_{\text{Pl}} \phi(0))\) .

\noindent
:

\be
\rho_\phi = \frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 \approx \frac{1}{2} \left(\frac{\beta
\rho_m}{M_{\text{Pl}} m_{\text{eff}}} \right)^2 \frac{e^{-2m_{\text{eff}} r}}{r^2}
\label{eq:app_rho2}
\ee

\noindent
\(r\) :

\be
\rho_\phi(r) \approx \frac{\rho_0}{1 + (m_{\text{eff}} r)^2}
\label{eq:app_cored}
\ee

\noindent
\(\rho_0 = (\beta \rho_m / M_{\text{Pl}} m_{\text{eff}})^2 / 2\). .

\subsection{ }
\label{app:screening}

\(\rho_c\) \(R\) \(\rho_\infty\) . :

\be
\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = m_{\text{eff}}^2(\rho) (\phi - \phi_{\min}(\rho))
\label{eq:app_screen1}
\ee

\noindent
(\(r < R\)) \(\rho = \rho_c\) \(\phi_{\min} = \phi_c\). \(\rho = \rho_\infty\) \(\phi_{\min} =
\phi_\infty\).

\noindent
 :

\be
\phi(r) = \phi_c + \frac{A}{r} \sinh(m_c r)
\label{eq:app_screen_in}
\ee

\noindent
 \(m_c = m_{\text{eff}}(\rho_c)\). :

\be
\phi(r) = \phi_\infty + \frac{B}{r} e^{-m_\infty (r - R)}
\label{eq:app_screen_out}
\ee

\noindent
 \(\phi\) \(\phi'\) \(r = R\):

\be
A = \frac{(\phi_\infty - \phi_c) m_\infty R e^{m_\infty R}}{m_\infty \cosh(m_c R) + m_c \sinh(m_c R)}
\label{eq:app_A}
\ee

\be
B = (\phi_\infty - \phi_c) \frac{m_c R \cosh(m_c R) - \sinh(m_c R)}{m_\infty \cosh(m_c R) + m_c \sinh(m_c R)}
\label{eq:app_B}
\ee

\noindent
 \(e^{-1}\) . (\(m_c R \gg 1\)) :

\be
r_{\text{screen}} \approx R + \frac{1}{m_\infty} \ln\left(\frac{\phi_\infty - \phi_c}{\beta M_{\text{Pl}} \Phi_N} \right)
\label{eq:app_rscreen}
\ee

\noindent
 \(\Phi_N = GM/R\) . \(r_{\text{screen}} \approx 10^3\) km.

\section{ }
\label{app:numerical}

\subsection{CLASS HLU}
\label{app:class}

  CLASS   :

\begin{enumerate}
   \item \textbf{:} \(w(z)\)   HLU (\ref{eq:w_of_rho}).
   \item \textbf{:} \(\mu(k, z)\)   (\ref{eq:growth}).
   \item \textbf{ :}   \(\delta\phi_k\)   .
\end{enumerate}

\subsection{ N-body}
\label{app:nbody}

 HLU Gadget-4   :

\begin{enumerate}
   \item   \(\phi(\mathbf{x})\).
   \item   : \(\mathbf{F} = -m \nabla (\Phi_N + \beta \phi / M_{\text{Pl}})\).
   \item   .
\end{enumerate}

\subsection{   }
\label{app:raytracing}

      :

\be
ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2
\label{eq:app_metric}
\ee

\noindent
 \(f(r) = 1 - 2G_{\text{eff}}(r) M/r\).   :

\be
G_{\text{eff}}(r) = G \left[1 + 2\beta^2 e^{-m_{\text{eff}}(r) r}\right]
\label{eq:app_Geff}

\ee

\noindent
.

\section{ }
\label{app:code}

:

\vspace{0.3cm}
\noindent
\textbf{}: \url{https://github.com/ZARKAM/HLU_COSMOLOGY}

\vspace{0.3cm}
\noindent
\textbf{ }:
\begin{itemize}
\item \texttt{hlu_core.py}: ( )
\item \texttt{hlu_class.py}: CLASS
\item \texttt{hlu_astro.py}: ( )
\item \texttt{hlu_lab.py}: ( )
\item \texttt{hlu_mcmc.py}: MCMC
\item \texttt{notebooks/}:
\end{itemize}

\vspace{0.3cm}
\noindent
\textbf{}:
\begin{itemize}
\item BAO DESI DR2
\item binned PR4 plik_lite
\item Pantheon+
\item M87* Sgr A* EHT
\item GWTC-4 LIGO-Virgo-KAGRA
\item NANOGrav
\end{itemize}

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\vspace{0.5cm}
\noindent
\textbf{ (HLU) . .}

\vspace{0.5cm}
\noindent
\textbf{ }

\end{document}
```
```